Unit :- 4 chapter- 7

Cartesian Product of sets, Relation, function and countable sets.

ORDERED PAIR :-

Example :- (H,H), (H,T), (T,H), (T,T)

Two ordered pairs (a_1, b_1) and (a_2, b_2) are said to be equal if and only if $a_1 = a_2$ and $b_1 = b_2$.

Ex :- 1 If the ordered pair $(x-2, 2y+1) = (y-1, x-2)$ are equal. find the value of x and y

Soln :-

$x-2 = y-1$
$x-2 = 1-1$
$x-2 = 0$
$x = 2$

$2y+1 = x-2$
$2y+1 = 0$
$2y+1 = 0$
$2y = 1$
$y = 1-2$
$y = -2$

By eq^n (1)

$x-2 = y-1$
$x-y = -1+2$
$\boxed{x-y=1}$ - - - - eq(1)

$2y+1 = x-2$
$2y-x = -2-1$
$2y-x = -3$ - - - - eq(2)

Eq(1) and eq(2) are elimination Method

$x - y = 1$
$-x + 2y = -3$
$y = -2$

putting the value of y in eq (1)

$x - y = 1$
$x - (-2) = 1$
$x + 2 = 1$
$x = 1 - 2$
$x = -1$

$x = -1 \quad y = -2$

Ans:-

CARTESIAN PRODUCT of two sets:-

Let A and B be two sets. The cartesian product of the set A and B is the set of all those order pair whose first co-ordinate is an element of A and B are co-ordinate is an element of B (in this order) and it is denoted by A x B.

$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$

$B \times A = \{(b, a) : b \in B \text{ and } a \in A\}$.

~~for example: If A = {1, 2, 3}, B = {2, 3, 4} & C = {3, 5} find (A x b) ∩ (A x C)~~

~~Soln:- A x b = {1, 2} x {2, 3}~~
~~(1, 2) (1, 3), (2, 2) (2, 3)~~
~~A x C = (1, 2) x (3, 5)~~
~~(1, 3), (1, 5); (2, 3) (2, 5)~~

If A = {1, 2} B = {2, 3} & C = {3, 5} find (A x b) ∩ (A x C).

Soln:-
$A \times B = \{1, 2\} \times \{2, 3\}$
$= (1, 2)\ (1, 3)\ (2, 2)\ (2, 3)$
$(A \times C) = \{1, 2\} \times \{3, 5\}$
$= (1, 3)\ (1, 5)\ (2, 3)\ (2, 5)$

$A\times B \cap A\times C = (1,3), (2,3)$

Ex:- 10 :- If A, B, C are any three non-empty sets then prove that $A\times(B\cup C) = (A\times B)\cup(A\times C)$

Soln :- let (x,y) be any arbitrary element of $A\times(B\cup C)$

$x \in A\times(B\cup C)$
$x\in A,\ y\in(B\cup C)$
$x\in A\ (y\in B \text{ or } y\in C)$
$x\in A,\ y\in B \text{ or } x\in A, y\in C$
$(x,y)(A\times B)\cup(A\times C) \ni (x,y)$
$(x,y)\in(A\times B)\cup(A\times C)$
$A\times(B\cup C)\subseteq(A\times B)\cup(A\times C)$ ... eq(1)

Again let
$(x',y')\in(A\times B)\cup(A\times C)$
$(x',y')\in(A\times B) \text{ or } (x',y')\in(A\times C)$
$(x'\in A, y'\in B) \text{ or } (x'\in A\ y'\in C)$
$x'\in A\ (y'\in B \text{ or } y'\in C)$
$x'\in A\ y'\in(B\cup C)$
$(x',y')\in A\times(B\cup C)$
$(A\times B)\cup(A\times C)\subseteq A\times(B\cup C)$ ... eq(2)

from eq(1) and eq(2)

$A\times(B\cup C) = (A\times B)\cup(A\times C)$

proved.

Chapter 7

Homework :- 11, 12

Ex:- 11 If A, B, C are any three non-empty sets, then prove that
$A\times(B\cap C) = (A\times B)\cap(A\times C)$

Soln :- let $(x,y)\in A\times(B\cap C)$

$x\in A,\ y\in(B\cap C)$
$x\in A\ (y\in B \text{ and } y\in C)$
$x\in A,\ y\in B \text{ and } x\in A\ y\in C$
$(x,y)\in(A\times B) \text{ and } (x,y)\in(A\times C)$
$(x,y)\in(A\times B)\cap(A\times C)$
$A\times(B\cap C)\subseteq(A\times B)\cap(A\times C)$ ... eq(1)

Again let
$(x',y')\in(A\times B)\cap(A\times C)$
$x'\in A\ y'\in B$ and $x'\in A,\ y'\in C$
$x'\in A\ (y'\in B \text{ and } y'=C)$
$(x'y')\in A(B\cap C)$
$(A\times B)\cap(A\times C)\subseteq A(B\cap C)$ ... eq(2)

from eq(1) and (2)

$A\times(B\cap C) = (A\times B)\cap(A\times C)$

Proved

Ex:- 12 If A, B, C are any three non-empty sets then prove that
$(A-B)\times C = (A\times C) - (B\times C)$

Soln:- Let $(x, y) \in (A-B)\times C$

$(x \in A$ and $x \notin B)\ y \in C$

$(x \in A, y \in C)$ and $(x \in B, y \in C)$

$(x, y)\ (A\times C) - (x, y)\ (B\times C)$

$(x, y)\ (A\times C) - (B\times C)$

$(x, y) \in (A\times C) - (B\times C)$.

$(A-B)\times C \in (A\times C) - (B\times C)$

$(A-B)\times C \subseteq (A\times C) - (B\times C)$ - - - eq (1)

Again let

$(x', y') \in (A\times C) - (B\times C)$

$(x' \in A$ and $y' \in C)$. $(x' \in B$ and $y' \in C)$

$x' y' \in C\ (x' \in A$ and $x' B)$

$(x' y') \in (A-B)\times C$

$(x' y') \subseteq (A-B)\times C$ - - - eq (2)

from eq (1) and (2)

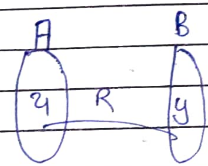
$(A-B)\times C = (A\times C) - (B\times C)$.

proved:-

Relation:-

In order to express a relation from the set A to set B. we always need a statement which connects the elements of A with the element of B.

for example:-



$R \subseteq A\times B$

not Empty $(\neq \phi)$

$R = \{(x, y) : x \in A, y \in B\}$

Binary:-

Types of Binary relation:-

1. reflexive relation:-

$\forall a \in A \Rightarrow (a, a) \in R$

$x \in A \Rightarrow (x, x) \in R$

2. Symmetric relation:-

$\forall a, b \in A$ $\boxed{aRb = bRa}$, $\forall a, b \in A$

reflexive, symmetric, transitive are equivalence relation.

3. Anti Symmetric relation :-

$\forall a, b \in A$ $\boxed{aRb,\ bRa \Rightarrow [a=b]}$

4. Transitive relation :-

$\forall a, b \in A$

$\boxed{aRb, bRc \Rightarrow aRc}$

Inverse relation :-

If $R \subseteq A \times B$ is a relation from the set A to the set B, then the inverse of R, denoted by R^{-1} is a relation from B to A.

$R^{-1} = \{(y, x);\ y \in B,\ x \in A\}$

Range of R^{-1} = Domain of R

$(a-c) = (a-b) + (b-c)$

$(a-c) = (a-c)$

Ex:-3 (a) Show that the relation $R = \{(a,b) : a-b =$ even integer and $a, b \in I\}$ in the set of I of integers is an equivalence relation.

Soln :- We known that R will be an equivalence relation if it is reflexive, symmetric and transitive.

(i) reflexive :-

$a \in I$ then $a - a = 0$ which is an even integer.

$(a, a) \in R \ \forall a \in I$

Hence R is a reflexive relation

(ii) Symmetric :- let $a, b \in I$

If $(a-b)$ is an even integer then $(b-a)$ is also an even integer

$\therefore (a,b) \in R = (b,a) \in R$ is true

Hence R is symmetric relation.

(iii) transitive :-

let $a, b, c \in I$

If $(a-b)$ and $(b-c)$ are even integers, th

$(a-c) = (a-b) + (b-c) =$ even integers

$(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$ true

Hence R is transitive relative

Since in the set I the relation R is reflexive symmetric and transitive and therefore R is an equivalence relation.

Ex:-6 If I is the set of non-zero integer and the relation R is define by xRy if $x^y = y^x$ where $x, y \in I$. then is the relation R an equivalence relation

Soln:- reflexivity :-

let $x \in I$

then xRx $\forall x \in I$, since x^x i certainly equal to x^x $(x^x = x^x)$

R is reflexive relation.

(ii) Symmetric

let $x, y \in I$

$xRy = yRx$ $\forall x y \in I$

$x^y = y^x \Rightarrow y^x = x^y$

$\because$ R is a symmetric relation

(iii) Transitive :-

let $x, y, z \in I$. and if xRy and yRz, then

$xRy, yRz \Rightarrow x^y = y^x, y^z = z^y$ ... eq(1)

Multiple z in both side

$(x^y)^z = (y^x)^z$

$x^{yz} = y^{xz}$ [By commutative law]

$(x^z)^y = (y^z)^x$ from eq(1)

$(x^z)^y = (z^y)^x$ [By commutative law]

$(x^z)^y = (z^x)^y$

$x^z = z^x$

$\Rightarrow xRz$

Hence R is a transitive relation.

Since the relation R is reflexive, symmetric and transitive, hence R is an equivalence relation.

Ex:-11 If R and S be a equivalence relation in the set X, then prove that $R \cap S$ is an equivalen relation in X

Soln:- (i) reflexivity :-

The show that are $R \cap S$ is an equivalence relation we have to verify the properties of reflexive, symmetric transitive.

(i) Reflexive :-

~~R is and S el~~
R & S and
$a \in X$, $(a,a) \in R$
$(a,a) \in R$ and $(a,a) \in S$
Hence
$a \in Y$
$(a,a) \in R \cap S$

(ii) Symmetry :-

$(a,b) \in R \Rightarrow (b,a) \in (b,a) \in R$
$(a,b) \in S \Rightarrow (b,a) \in S$
$a, b \in Y$
$(a,b) \in R \cap S \Rightarrow (a,b) \in R$ and $(a,b) \in S$
$= (b,a) \in R$ and $(b,a) \in S$
$= (b,a) \in R \cap S$

$R \cap S$ is symmetric

(iii) Transitivity :-

$(a,b) \in R \Rightarrow (b,c) \in R \Rightarrow (a,c) \in R$
$(a,b) \in S \Rightarrow (b,c) \in S \Rightarrow (a,c) \in S$
$a, b, c \in Y$

$(a,b,c) \in R \cap S \Rightarrow (a,b)(b,c) \in R$ and $(a,c) \in S$
(a,b)

$\Rightarrow (a,b)(b,c) \in R$ and $(a,b)(b,c) \in S$
$\Rightarrow (a,c) \in R$ and $(c,a) \in R$
$\Rightarrow (a,c) \in R \cap S$

$R \cap S$ is transitivity.

Hence $R \cap S$ is equivalent relation in X.

Ex. 13 If R is an equivalence relation in the set A. then prove that equivalence relation in the set A.

Soln :- (i) Reflexive :-
$a \in A \Rightarrow (a,a) \in R$
$(a,a) \in R \Rightarrow (a,a) \in R^{-1}$

R^{-1} is reflexive

(ii) Symmetric :-

$(a,b) \in R \Rightarrow (b,a) \in R^{-1}$
$(b,a) \in R \Rightarrow (a,b) \in R^{-1}$
$(b,a) \in R^{-1} \Rightarrow (a,b) \in R^{-1}$

R^{-1} is symmetric

(iii) transitivity :-

$(a, b, c) \in A$

$(a, b) \in R \Rightarrow (b, c) \in R \Rightarrow (a, c) \in R^{-1}$

$(a, b) \in R, (b, c) \in R \Rightarrow (a, c) \in R$

$(c, a) \in R^{-1}$

$(c, b) \in R^{-1}$ and $(b, a) \in R^{-1} \Rightarrow (c, a) \in R^{-1}$

R^{-1} is transitive

Since R^{-1} is reflexive symmetric and transitive in the set A, therefore R^{-1} is an equivalence relation.