From a point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ perpendiculars are drawn on the axes and join the foots of the perpendiculars. Show that the straight lines so drawn always touches the curve .

OR

â§ýyã âØàsk ABC **t¦ §ýà £0° j™¦ tàÂà Öààm §ýLâk¥ ñ vªààÂk âwâo ZàuQý §ýLâk¥ ñ**

Find the maximum value of in a tringle ABC. Use Lagrange's method.

ZàÎÂà-5. ytà§ýv §ýà tÌuà§ýÂà §ýLâk¥ kÑà¦ R **Àãiwùà y¦ qáÊr÷ ÕàØàÄýv Ñ¦ ñ**

Evaluate the integral where R is the region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

OR

âÂàÈÂàâvâhm ytà§ýv §ý §ýt §ýà qáÊwâmm §ýÊm Ñ¥ ytà§ýv §ýà tàÂà Öààm §ýLâk¥ ß

Change the order of integration and evaluate the following integral :

----x----

Roll No......................

Total No. of Section : 03

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Annual Examination - 2017

B.Sc.-II

MATHEMATICS

Paper - I

ADVANCED CALCULUS

Max.Marks : 50

Time : 3 Hrs. **Min Marks : 17**

¹âq ß h½»¦ '¡' t¦ Ày ¡âmvi ÙàÊã ZàÎÂà Ñ¦, âkÂÑ¦ Ñv §ýÊÂàà ¡âÂàwàu Ñ¦ ñ h½»¦ 'r' t¦ v i ÙàÊã ZàÎÂà h½»¦ 'y' t¦ Àã i £ÙàÊãu ZàÎÂà Ñ¦ ñ h½»¦ '¡' §ýà yry qÑv Ñv §ýÊ¦ ñ

Note : Section 'A' is objective type, containing 10 questions, is compulsory. Section 'B' consists of short answer type questions and Section 'C' consists of long answer type questions. Section 'A' has to be solved first.

cos A cos B cos C dx dy

h½»¦-'¡' **(Section-'A')**

âÂàÈÂàâ§ým ZàÎÂàà¦ §ý £ÙàÊ Ààâk¥ ñ (Answer the following questions.) (1x10=10)

ZàÎÂà-1. hàvã Ðnàà sáÊ¥ (Fill in the blanks**) ß**

$$\lim_{n\to\infty}\left(\frac{1}{3}+\frac{1}{3^2}+---+\frac{1}{3^n}\right)=\ldots\ldots\ldots\ldots\ldots$$

ZàÎÂà-2. yÑã âw§ýÌq jâÂà¥ :

ó½ãà $\sum\frac{n^{n^2}}{(1+n)^{n^2}}$ **¡âsyàÊã/¡qyàÊã Ñ¦ ñ**

Choose the correct option ß

The series $\sum\frac{n^{n^2}}{(1+n)^{n^2}}$ is convergent/divergent.

P.T.O.

ZàÎÀà-3. Äÿ**v**Âà $f(x) = x - |x| \; \forall \; x \in IR$ §ÿ âv¥ $f(0-0) = ?$, $f(0+0) = ?$

For the function

$$f(0-0) = ? \;,\; f(0+0) = ?$$

ZàÎÀà-4. Äÿ**v**Âà $x = 0$ ¡w§ÿ**v**Ââàu Ñ uâÀ

.

Function is differentiable at $x = 0$ if

.

ZàÎÀà-5. §ÿà ¡ÛÐm½w Ñ / ¡ÛÐm½w ÀàÑã Ñ ñ

$\lim_{(x,y)\to(0,0)} \frac{2y}{x}$ exists/doesnot exists.

ZàÎÀà-6. uâÀ $f_x(a,b) = \lim_{h\to 0} \frac{f(a+h,b) - f(a,b)}{h}$ ¡àÊ

$f_y(a,b) = \lim_{k\to 0} \frac{f(a,b+k) - f(a,b)}{k}$ $f_{yy}(a,b) = ?$

If and

$f_y(a,b) = \lim_{k\to 0} \frac{f(a,b+k) - f(a,b)}{k}$ then $f_{yy}(a,b) = ?$.

OR

ây÷ §ÿLâk¥ â§ÿ (Prove that) :

kÑà $\theta, -1$ ¡àÊ §ÿ rä**j** §ÿà¢ y©uà Ñ ñ

where is a number between -1 and 1.

ZàÎÀà-3. uâÀ kÑà $r^2 = x^2 + y^2$ mà ÀÎàà¢u â§ÿ ß

If where $r^2 = x^2 + y^2$ then show that :

OR

$F'(x+\theta h)$ … $(v-w)(w-u)$ … $0,$ if $x = 0$

$u^3 + v^3 + w^3 = x + y + z \,, u^2 + v^2 + w^2 = x^3 + y^3 + z^3 \,, u + v + w = x^2$

mà ây÷ §ÿÊà â§ÿ (Then prove that) :

ZàÎÀà-4. Àãiwûà §ÿ â§ÿyã ârÂÀ y ¡Õàà qÊ vÈr »àv kàm Ñ ¡àÊ ¢Âà vÈrà §ÿ qàÀà §ÿà âtvàuà kàmà Ñ ñ ây÷ §ÿLâk¥ â§ÿ ¢y Zà§ÿâÊ ZààÄm yÊv Êhà¥ ÑtÎàà w§ÿ §ÿà ÐqÎà §ÿÊmã Ñ ñ

h½»|-'y' (Section-'C')

âÂàÈÂààlâ§ým ZàÎÂàà¡ §¡ý £ÙàÊ Ààâk¥ (Answer the following questions) : **(5x5=25)**

ZàÎÂà-1. âý÷ §ýLâk¥ â§ý ¡Ââ§ýt $\left\{\frac{(3n)!}{(n!)^3}\right\}^{1/n}$ ¡âsyàÊã Ñ ñ

Prove that the sequence is convergent.

OR

âÂàÈÂààvâhm ó½ãã §ýL ¡âsyàâÊmà §ýà qÊãÕà½à §ýLâk¥ ß

Test the convergence of the following series :

ZàÎÂà-2. âÂàÈÂààvâhm Äývâà §ý âv¥ ¡ÂmÊàv t¦ Äývâà §ý yàm¼u ¡àÊ ¡w§ývÂààumà §ýL âwwjÂàà §ýLâk¥ :

Discuss the continuity and differentiability of the following function in the interval :

ZàÎÂà-7. uâÀ α w§ý§ýv §ýà Zààjv Ñ mà ¢y w§ý§ýv §ýà ¡Âwàvàq .... Ñàªàà ñ

If is the parameter of family of curves . Then the equation of envelope for the family is ....

ZàÎÂà-8. Äývâà §ýà £0° j™¦ uà ââ0ÈÂà™¦ tàÂà .... Ñàªàà ñ

Maximum or minimum value of function is ....

ZàÎÂà-9. §ýà tàÂà β Äývâà §ý Ûýq t¦ Ñàªàà .... ?

in terms of β functions is ....

ZàÎÂà-10.

$x\cos\alpha + y\sin\alpha = l\sin\alpha\cos\alpha$

$x > 0$

$\int_0^1 x^2(1-x)^3\,dx = ?$

$f(x) = \begin{cases} \ldots & \text{kr (when) } 0 \le x \le 1 \\ 2x^2 - 3x + \frac{3}{2} & \text{kr (when) } 1 \le x \le 2 \end{cases}$

h½»|-'r' (Section-'B')

âÂàÈÂààlâ§ým ZàÎÂàà¡ §¡ý £ÙàÊ Ààâk¥ ñ (Answer the following questions.) **(3x5=15)**

ZàÎÂà-1. §ýàÎàã ¡Ââ§ýt §ýL qáÊsàxà £ÀàÑÈ½à yâÑm Àà¡ ñ

Define cauchy sequence and give an example of it.

OR

âÂàÈÂààvâhm ó½ãã §ýL ¡âsyàâÊmà §ýà qÊãÕà½à §ýLâk¥ ß

Test the convergence of the following series :

$$\sqrt{\frac{1}{2^3}} + \sqrt{\frac{2}{3^3}} + \sqrt{\frac{3}{4^3}} + \ldots\ldots\ldots\ldots\ldots$$

P.T.O.

ZàÎÀà-2. i) ÊàvÌ ZàtÌu ¥wÌ ii) ¹ÌvÊ ZàtÌu §ýà §ýnÀà Ààâk¥ ñ

Give the statement of i) Rolle's theorem and ii) Taylor's theorem.

OR

ÀÎàà¢uÌ â§ý âÂàÈÂààvâhm ÄývÀà §ý âv¥ $f'(0)$ §ýà ¡ÌÐmÌ¼w ÀàÑã Ñ ß

Show that $f'(0)$ does not exists for the following function :

$f(x) =$ [redacted]

ZàÎÀà-3. tàÂàà $f : R^3 \to R$ âÂàÈÂà Zà§ýàÊ yÌ qáÊsààxm Ñ mà $\lim_{(x,y)\to(0,0)} f(x,y)$

Öààm §ýLâk¥ ß

Let $f : R^3 \to R$ be defined as follows then find $\lim_{(x,y)\to(0,0)} f(x,y)$:

$f(x,y) =$ [redacted]

OR

uâÀ $u = \log\left(\dfrac{x^4 + y^4}{x + y}\right)$ mà ÀÎàà¢uÌ â§ý ß

If $u = \log\left(\dfrac{x^4 + y^4}{x + y}\right)$ then show that :

$x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 3$

ZàÎÀà-4. ¡âmqÊwvu $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ §ý §ýÂôk §ýà ytã§ýÊ½à Öààm §ýLâk¥ ñ

Find the equation of evolute for the hyperbola .

OR

¥§ý âØàsÌk §ý ¡ÌÀÊ ¥§ý ârÂÀÌ ¢y Zà§ýàÊ Öààm §ýLâk¥ â§ý mãÂààÌ §ýà½ààu ârÂÀÌ ¡Ìà yÌ ¢y§ýL ÀáÊuàÌ §ý wªààÌ §ýà uàªà âÂàÒÈÂà™Ì Ñ ñ

Find a point inside a triangle so that the sum of squares of its distance from all three angular points is minimum.

ZàÎÀà-5. âÂàÈÂààvâhm §ýà tàÂà Öààm §ýLâk¥ ß

Evaluate :

OR

tÌÌuà§ýÂà §ýLâk¥ ß

Evaluate :